TFS High School

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COURSE OUTLINE

Calculus and Vectors 12

MCV4U (University)

Department	Mathematics
Instructor	Mr. Hamid Rajaee
Course Development Date	September 2012
Ministry Course Code	MCV4U
Credit Value	1.00
Ministry Curriculum Document	Mathematics, The Ontario Curriculum, Grades 11 and 12, 2007 (Revised) http://www.edu.gov.on.ca/eng/curriculum/secondary/math1112currb.pdf http://www.edu.gov.on.ca/eng/policyfunding/growsuccess.pdf
Prerequisites	Grade 12 Advanced Functions, University, must be taken prior to or concurrently with Calculus and Vectors
Course Revision Date (TFS)	September 2020

Course Description:

This course builds on students' previous experience with functions and their developing understanding of rates of change. Students will solve problems involving geometric and algebraic representations of vectors and representations of lines and planes in three dimensional space; broaden their understanding of rates of change to include the derivatives of polynomial, sinusoidal, exponential, rational, and radical functions; and apply these concepts and skills to the modelling of real-world relationships. Students will also refine their use of the mathematical processes necessary for success in senior mathematics. This course is intended for students who choose to pursue careers in fields such as science, engineering, economics, and some areas of business, including those students who will be required to take a university-level calculus, linear algebra, or physics course.

Overall Expectations - MCV4U

RATE OF CHANGE	
Overall Expectations	
110.050.01.01	demonstrate an understanding of rate of change by making connections between average rate of change over an interval and instantaneous rate of change at a point, using the slopes of secants and tangents and the concept of the limit;
110.050.01.02	graph the derivatives of polynomial, sinusoidal, and exponential functions, and make connections between the numeric, graphical, and algebraic representations of a function and its derivative;
110.050.01.03	verify graphically and algebraically the rules for determining derivatives; apply these rules to determine the derivatives of polynomial, sinusoidal, exponential, rational, and radical functions, and simple combinations of functions; and solve related problems.
DERIVATIVES AND THEIR APPLICATIONS	
Overall Expectations	
110.050.02.01	make connections, graphically and algebraically, between the key features of a function and its first and second derivatives, and use the connections in curve sketching;
110.050.02.02	solve problems, including optimization problems, that require the use of the concepts and procedures associated with the derivative, including problems arising from real-world applications and involving the development of mathematical models.
GEOMETRY AND ALGEBRA OF VECTORS	
Overall Expectations	
110.050.03.01	demonstrate an understanding of vectors in two-space and three-space by representing them algebraically and geometrically and by recognizing their applications;
110.050.03.02	perform operations on vectors in two-space and three-space, and use the properties of these operations to solve problems, including those arising from real-world applications;

110.050.03.03	distinguish between the geometric representations of a single linear equation or a system of two linear equations in two-space and three-space, and determine different geometric configurations of lines and planes in three-space;
110.050.03.04	represent lines and planes using scalar, vector, and parametric equations, and solve problems involving distances and intersections.

Units of Study

Unit	Titles and Descriptions	Time and Sequence
Part One	**The Geometry and Algebra of Vectors**	
Unit 1	**Vectors** There are four main topics pursued in this initial unit of the course. These topics are: an introduction to vectors and scalars, vector properties, vector operations and plane figure properties. Students will tell the difference between a scalar and vector quantity, they will represent vectors as directed line segments and perform the operations of addition, subtraction, and scalar multiplication on geometric vectors with and without dynamic geometry software. Students will conclude the first half of the unit by proving some properties of plane figures, using vector methods and by modeling and solving problems involving force and velocity. Next students learn to represent vectors as directed line segments and to perform the operations of addition, subtraction, and scalar multiplication on geometric vectors with and without dynamic geometry software. The final topic involves students in proving some properties of plane figures using vector methods.	13 hours
Unit 2	**Vector Applications** Cartesian vectors are represented in two-space and three-space as ordered pairs and triples, respectively. The addition, subtraction, and scalar multiplication of Cartesian vectors are all investigated in this unit. Applications involving work and torque are used to introduce and lend context to the dot and cross products of Cartesian vectors. The vector and scalar projections of Cartesian vectors are written in terms of the dot product. The properties of vector products are investigated and proven. These vector products will be revisited to predict characteristics of the solutions of systems of lines and planes in the intersections of lines and planes.	15 hours
Unit 3	**Intersection of Lines and Planes** This unit begins with students determining the vector, parametric and symmetric equations of lines in R2 and R3 . Students will go on to determine the vector, parametric, symmetric and scalar equations of planes in 3-space. The intersections of lines in 3-space and the intersections of a line and a plane in 3-space are then taught. Students will learn to determine the intersections of two or three planes by setting up and solving a system of linear equations in three unknowns. Students will interpret a system of two linear equations in two unknowns geometrically, and relate the geometrical properties to the type of solution set the system of equations possesses. Solving problems involving the intersections of lines and planes, and presenting the solutions with clarity and justification forms the next challenge. As work with matrices continues students will define the terms related to matrices while adding, subtracting, and multiplying them. Students will solve systems of linear equations involving up to three unknowns, using row reduction of matrices, with and without the aid of technology and interpreting row reduction of matrices as the creation of new linear systems equivalent to the	19 hours

	original constitute the final two new topics of this important unit.	
Part Two	**Calculus and Rates of Change**	
Unit 4	**Concepts of Calculus** A variety of mathematical operations with functions are needed in order to do the calculus of this course. This unit begins with students developing a better understanding of these essential concepts. Students will then deal with rates of change problems and the limit concept. While the concept of a limit involves getting close to a value but never getting to the value, often the limit of a function can be determined by substituting the value of interest for the variable in the function. Students will work with several examples of this concept. The indeterminate form of a limit involving factoring, rationalization, change of variables and one sided limits are all included in the exercises undertaken next in this unit. To further investigate the concept of a limit, the unit briefly looks at the relationship between a secant line and a tangent line to a curve. To this point in the course students have been given a fixed point and have been asked to find the tangent slope at that value, in this section of the unit students will determine a tangent slope function similar to what they had done with a secant slope function. Sketching the graph of a derivative function is the final skill and topic.	18 hours
Unit 5	**Derivatives** The concept of a derivative is, in essence, a way of creating a short cut to determine the tangent line slope function that would normally require the concept of a limit. Once patterns are seen from the evaluation of limits, rules can be established to simplify what must be done to determine this slope function. This unit begins by examining those rules including: the power rule, the product rule, the quotient rule and the chain rule followed by a study of the derivatives of composite functions. The next section is dedicated to finding the derivative of relations that cannot be written explicitly in terms of one variable. Next students will simply apply the rules they have already developed to find higher order derivatives. As students saw earlier, if given a position function, they can find the associated velocity function by determining the derivative of the position function. They can also take the second derivative of the position function and create a rate of change of velocity function that is more commonly referred to as the acceleration function which is where this unit ends.	13 hours
Unit 6	**Curve Sketching** In previous math courses, functions were graphed by developing a table of values and smooth sketching between the values generated. This technique often hides key detail of the graph and produces a dramatically incorrect picture of the function. These missing pieces of the puzzle can be found by the techniques of calculus learned thus far in this course. The key features of a properly sketched curve are all reviewed separately before putting them all together into a full sketch of a curve.	7 hours
Unit 7	**Derivative Applications and Related Rates** A variety of types of problems exist in this unit and are generally grouped into the following categories: Pythagorean Theorem Problems (these include ladder and intersection problems), Volume Problems (these usually involve a 3-D shape being filled or emptied), Trough Problems, Shadow problems and General Rate Problems. During this unit students will look at each of these types of problems individually.	8 hours
Unit 8	**Derivative of Exponents and Log Functions-Exponential Functions** This unit begins with examples and exercises involving exponential and logarithmic functions using Euler's number (e). But as students have already seen, many other bases exist for	7 hours

	exponential and logarithmic functions. Students will now look at how they can use their established rules to find the derivatives of such functions. The next topic should be familiar as the steps involved in sketching a curve that contains an exponential or logarithmic function are identical to those taken in the curve sketching unit studied earlier in the course. Because the derivatives of some functions cannot be determined using the rules established so far in the course, students will need to use a technique called logarithmic differentiation which is introduced next.	
Unit 8	**Trig Differentiation and Application** A brief trigonometry review kicks off this unit. Then students turn their attention to special angles and the CAST rule which has been developed to identify which of the basic trigonometric ratios is positive and negative in the four quadrants. Students will then solve trigonometry equations using the CAST rule to locate other solutions. Two fundamental trigonometric limits are investigated for the concepts of trigonometric calculus to be fully understood. The unit ends, as in all other units in the course, with an assignment and a unit quiz.	8 hours
	Final Evaluation	2 hours
	Total	**110 hours**

Assessment and Evaluation Strategies of Student Performance based on Growing Success

Strategy	Purpose	Who	Assessment Tool
Self Assessment Quizzes	Diagnostic	Self/Teacher	Marking scheme
Problem Solving	Diagnostic	Self/Peer/Teacher	Marking scheme
Graphing Application	Diagnostic	Self	Anecdotal records
Problem Solving	Assessment	Peer/teacher	Marking scheme
Research	Assessment	Peer/teacher	Anecdotal records
Problem Solving	Evaluation	Teacher	Marking scheme
Graphing	Evaluation	Teacher	Checklist
Investigations	Evaluation	Teacher	Checklist
Unit Tests	Evaluation	Teacher	Marking scheme
Final Exam	Evaluation	Teacher	Checklist

Teaching / Learning Strategies:

Since the over-riding aim of this course is to help students use language skillfully, confidently and flexibly, a wide variety of instructional strategies are used to provide learning opportunities to accommodate a variety of learning styles, interests and ability levels. These include:

Guided Exploration	Problem Solving	Graphing
Visuals	Direct Instruction	Independent Reading
Independent Study	Ideal Problem Solving	Multimedia Productions
Logical Mathematical Intelligence	Graphing Applications	Problem Posing
Model Analysis		Self-Assessments

ASSESSMENT AND EVALUATION- GROWING SUCCESS

Diagnostic assessment is used at the beginning of a unit to assist in determining a starting point for instruction. **Growing Success** ensures well-rounded, ongoing assessment of student learning. Assessment for Learning (AFL) provides information to students as they are learning and refining their skills. Assessment as Learning (AAL) acts as a stepping-stone for students to begin applying their understanding using critical thinking; it bridges the gap between AFL and AOL. Assessment of Learning (AOL), at the end of units and course, provides students with the opportunity to synthesize/apply/demonstrate their learning and the achievement of the expectations. The following is a list of specific assessment/evaluation strategies that the teacher may use but is not limited to the items shown.

Term work: 70%	**15%** Knowledge & Understanding: subject-specific content acquired (knowledge), and the comprehension of its meaning and significance (understanding). **30%** Application: the use of knowledge and skills to make connections within and between various contexts. **35%** Thinking: the use of critical and creative thinking skills and/or processes. **20%** Communication: the conveying of meaning through various forms (oral, visual, and/or written).
Final Exam: 30%	**30%** Exam (1 hour exam within a 1.5 hour time slot) - consisting of a variety of question types (e.g., short answer, multiple choice, extended response, problem solving, etc.); completed during exam time period; individual student effort; evaluated by teacher
Your final grade will be calculated by combining your Term (70%) grade and your Exam and Performance Task Evaluations (30%).	

Potential Resources:

- MCV4U online course of study
- McGraw-Hill Ryerson Calculus and Vectors 12, Speijer, Jacob, Wayne Erdman, David Petro et al., McGraw Hill Ryerson, 2008
- Nelson Calculus and Vectors, Kirkpatrick, Chris, Peter Crippin, Robert Donato et al., Nelson Education Ltd., 2009

Achievement Chart: Mathematics, Grades 9-12

Categories	50-59% (Level 1)	60-69% (Level 2)	70-79% (Level 3)	80-100% (Level 4)
Knowledge and Understanding - Subject-specific content acquired in each course (knowledge), and the comprehension of its meaning and significance (understanding)				
	The student:			
Knowledge of content (e.g., facts, terms, definitions)	demonstrates limited knowledge of content	demonstrates some knowledge of content	demonstrates considerable knowledge of content	demonstrates thorough knowledge of content
Understanding of content (e.g., concepts, ideas, theories, procedures, processes, methodologies, and/or technologies)	demonstrates limited understanding of content	demonstrates some understanding of content	demonstrates considerable understanding of content	demonstrates thorough and insightful understanding of content
Thinking - The use of critical and creative thinking skills and/or processes				
	The student:			

Use of planning skills (e.g., focusing research, gathering information, organizing an inquiry, asking questions, setting goals)	uses planning skills with limited effectiveness	uses planning skills with moderate effectiveness	uses planning skills with considerable effectiveness	uses planning skills with a high degree of effectiveness
Use of processing skills (e.g., inquiry process, problem-solving process, decision-making process, research process)	uses processing skills with limited effectiveness	uses processing skills with some effectiveness	uses processing skills with considerable effectiveness	uses processing skills with a high degree of effectiveness
Use of critical/creative thinking processes (e.g., oral discourse, research, critical analysis, critical literacy, metacognition, creative process)	uses critical / creative thinking processes with limited effectiveness	uses critical / creative thinking processes with some effectiveness	uses critical / creative thinking processes with considerable effectiveness	uses critical / creative thinking processes with a high degree of effectiveness
Communication - The conveying of meaning through various forms				
	The student:			
Expression and organization of ideas and information (e.g., clear expression, logical organization) **in oral, graphic, and written forms, including media forms**	expresses and organizes ideas and information with limited effectiveness	expresses and organizes ideas and information with some effectiveness	expresses and organizes ideas and information with considerable effectiveness	expresses and organizes ideas and information with a high degree of effectiveness
Communication for different audiences (e.g., peers, adults) **and purposes** (e.g., to inform, to persuade) **in oral, written, and visual forms**	communicates for different audiences and purposes with limited effectiveness	communicates for different audiences and purposes with some effectiveness	communicates for different audiences and purposes with considerable effectiveness	communicates for different audiences and purposes with a high degree of effectiveness
Use of conventions (e.g., conventions of form, map conventions), **vocabulary, and terminology of the discipline in oral, written, and visual forms**	uses conventions, vocabulary, and terminology of the discipline with limited effectiveness	uses conventions, vocabulary, and terminology of the discipline with some effectiveness	uses conventions, vocabulary, and terminology of the discipline with considerable effectiveness	uses conventions, vocabulary, and terminology of the discipline with a high degree of effectiveness
Application - The use of knowledge and skills to make connections within and between various contexts				
	The student:			
Application of knowledge and skills (e.g., concepts, procedures, processes, and/or technologies) **in familiar contexts**	applies knowledge and skills in familiar contexts with limited effectiveness	applies knowledge and skills in familiar contexts with some effectiveness	applies knowledge and skills in familiar contexts with considerable effectiveness	applies knowledge and skills in familiar contexts with a high degree of effectiveness
Transfer of knowledge and skills (e.g., concepts, procedures, methodologies, technologies) **to new contexts**	transfers knowledge and skills to new contexts with limited effectiveness	transfers knowledge and skills to new contexts with some effectiveness	transfers knowledge and skills to new contexts with considerable effectiveness	transfers knowledge and skills to new contexts with a high degree of effectiveness
Making connections within	makes connections	makes connections	makes connections	makes connections

and between various contexts (e.g., past, present, and future; environmental; social; cultural; spatial; personal; multidisciplinary)	within and between various contexts with limited effectiveness	within and between various contexts with some effectiveness	within and between various contexts with considerable effectiveness	within and between various contexts with a high degree of effectiveness

Learning Skills:

Learning Skills are skills and habits are essential to success in school and in the workplace. Teachers report achievement on the six Learning Skills in the table below using letter codes:

E = Excellent G = Good S = Satisfactory N = Needs Improvement.

Learning Skills	Sample Behaviors
Responsibility	The student fulfils responsibilities and commitments within the learning environment; completes and submits class work, homework, and assignments according to agreed-upon timelines; takes responsibility for and manages own behavior.
Organization	The student devises and follows a plan and process for completing work and tasks; establishes priorities and manages time to complete tasks and achieve goals; identifies, gathers, evaluates, and uses information, technology, and resources to complete tasks.
Independent Work	The student independently monitors, assesses, and revises plans to complete tasks and meet goals; uses class time appropriately to complete tasks; follows instructions with minimal supervision.
Collaboration	The student accepts various roles and an equitable share of work in a group; responds positively to the ideas, opinions, values, and traditions of others; builds healthy peer-to-peer relationships through personal and media-assisted interactions; works with others to resolve conflicts and build consensus to achieve group goals; shares information, resources, and expertise and promotes critical thinking to solve problems and make decisions.
Initiative	The student looks for and acts on new ideas and opportunities for learning; demonstrates the capacity for innovation and a willingness to take risks; demonstrates curiosity and interest in learning; approaches new tasks with a positive attitude; recognizes and advocates appropriately for the rights of self and others.
Self-Regulation	The student sets own individual goals and monitors progress towards achieving them; seeks clarification or assistance when needed; assesses and reflects critically on own strengths, needs, and interests; identifies learning opportunities, choices, and strategies to meet personal needs and achieve goals; perseveres and makes an effort when responding to challenges.

Academic Honesty: Cheating and Plagiarism:

Plagiarism is a serious offense. It is defined as taking words, phrasing, sentence structure, or any other element of the expression of another person's **ideas**, and using them as if they were your own. Plagiarism is a violation of another person's rights, whether the material taken is excessive or small. Students will be assisted in developing strategies and techniques to avoid plagiarism. They need to be aware that plagiarized term work will be penalized and could result in a mark of zero.

Assignment Policy, Missed Tests and Attendance: Please refer to Student Contract Form

Program Planning Considerations:

Program Planning Considerations for English: Teachers who are planning a program in English must take into account considerations in a number of important areas. Essential information that pertains to all disciplines is provided in the companion piece to this document, The Ontario Curriculum, Grades 9 to 12: Program Planning and Assessment, 2007 (Revised).

The areas of concern to all teachers that are outlined there include the following:

- Education for Exceptional Students
- The Role of Technology in the Curriculum √
- English as a Second Language (ESL) √
- English as a Second Language (ESL) and English Literacy Development (ELD)
- Antidiscrimination Education in the English Program √
- Literacy, Numeracy, and Inquiry/Research Skills √
- Career Education √
- Cooperative Education •
- Health and Safety